# Sustainable Underwater Robotic Networks using Autonomous Mobile Wireless Energy Transfer

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Abstract—The exploration and exploitation of ocean resources are labor-intensive and dangerous for human beings. The adoption of autonomous robots provides an efficient and safer solution. However, a single underwater robot has limited energy and communication capability which restrict its operating duration and mission range. The underwater cyber-physical system using swarm robotics is highly desirable where robots move cooperatively to accomplish tasks that cannot be done by any individual. In this paper, an autonomous mobile wireless charger is proposed to be used in order to recharge mobile robots to overcome the energy limitation and further extend the mission duration. First, the feasibility of underwater wireless energy transfer is proved and its achievable efficiency is obtained. Then, a model is developed to reveal the interplay among the communication, the wireless energy transfer, and the optimal motion control. Our results show that the trajectory tracking can significantly affect wireless energy transfer efficiency. Furthermore, the optimal trajectory for the mobile charger depends on the swarm size and dynamics.

*Index Terms*—Robotic networks, swarm robotics, underwater, wireless communication, wireless energy transfer.

#### I. INTRODUCTION

Underwater cyber-physical systems consisting of a swarm of mobile Autonomous Underwater Vehicles (AUVs) have a large number of civilian and military applications. AUVs are networked by using wireless underwater communications and they have the capabilities of computing, sensing, locomotion, and object detection [1]. Different from the remotely operated underwater vehicles which use umbilical cables to provide power and data communication, AUVs are fully autonomous and they do not require external power supplies or human intervention. Due to their limited battery energy, AUVs have a very limited operating range and mission duration which cannot provide persistent sensing or target tracking.

In this paper, we propose to use a mobile charger to wirelessly recharge mobile AUVs which aims to enable sustainable underwater cyber-physical systems with unlimited operating range and mission duration. The Wireless Energy Transfer (WET) leverages magnetic induction which has a small propagation loss in underwater due to its long wavelengths [1]. We consider AUVs are mission-driven and they cannot stop or return to be recharged. Their trajectories are determined by swarm dynamics such as wireless network connectivity, obstacles, and many other factors. AUVs can cooperate with each other to find optimal trajectories. This paper focuses on the interactions between AUVs and the mobile charger, and we develop an optimal trajectory for the mobile charger to maximize the WET efficiency. An illustration of the proposed system is shown in Fig. 1. AUVs send their location information to the mobile charger upon which the mobile charger



Fig. 1. Illustration of underwater mobile charging. A mobile charger is charging two mobile AUVs simultaneously.

estimates AUVs motion trajectory and plans its own motion to achieve high WET efficiency.

However, the trajectory design is not trivial due to the following challenges. First, existing underwater localization algorithms are not perfect and thus the locations of AUVs are not accurate, which may provide misleading information for the mobile charger. Second, the mobile charger has to move along an optimal trajectory to maximize WET efficiency while avoiding collisions, which is different from static WET. Third, when there are multiple AUVs, the mobile charger has to consider WET efficiency and fairness jointly. Currently, in the literature, there is no effort to address the above challenges for underwater cyber-physical systems.

In this paper, we consider the effect of localization error on mobile WET. The mobile charger does not have perfect location information of AUVs which is a common problem in harsh underwater environments. The mobile charger uses a Kalman Filter to track and predict the locations of AUVs upon which it plans its trajectory. We first review the characteristics of the underwater WET channel by considering the surface lateral waves. Then, we study the interactions between a single AUV and the mobile charger to obtain the optimal trajectory for the mobile charger and prove the feasibility. After that, we develop the mobile WET for multiples AUVs by considering fairness, energy consumption, and battery level. The developed solution is a nonconvex problem and we obtain an approximation of the solution. Also, we show the optimal mobile charger's trajectory highly depends on the distance between AUVs. The proposed approach is evaluated using numerical simulations.

#### II. MOBILE CHARGER TRAJECTORY PLANNING

In this section, we introduce the AUV location and motion model. Then, we present the WET channel model. After that, we study the optimal trajectory of the mobile charger for recharging a single AUV and multiple AUVs. Let the accurate location of an AUV be  $x^r(t_i) \in \mathbb{R}^3$ , for  $i = 0, 1, \cdots$ , where  $t_0$  is the starting time, and  $\Delta t = t_{i+1} - t_i$  which is the time interval. The AUV's motion is governed by

$$s^{r}(t_{i}) = As^{r}(t_{i-1}) + w(t_{i-1}),$$
 (1)

where  $s^r(t_i)^T = [x^r(t_i), v^r(t_i)]^T$ ,  $v^r(t_i) \in \mathbb{R}^3$  is the velocity of the AUV in the 3D space,  $A \in \mathcal{R}^{6 \times 6}$  is the transition matrix which relates the AUV's current state with the next state, and  $w(t_i) \sim \mathcal{N}(0, Q)$  is Gaussian noises with covariance matrix Q. Due to the underwater localization errors, the AUV considers its location as

$$\boldsymbol{y}^{r}(t_{i}) = \boldsymbol{H}\boldsymbol{s}^{r}(t_{i}) + \boldsymbol{v}(t_{i}), \qquad (2)$$

where  $\boldsymbol{H} \in \mathcal{R}^{3 \times 6}$  relates the state to the measurement, and  $\boldsymbol{v}(t_i) \sim \mathcal{N}(0, \boldsymbol{R})$  is the measurement Gaussian noises with covariance matrix  $\boldsymbol{R}$ . Once the connection between the mobile charger and an AUV is built, the AUV sends  $\boldsymbol{y}^r(t_i)$ to the mobile charger every  $\Delta t$ . Assume that the wireless communication is reliable and there is no error, the mobile charger receives  $\boldsymbol{y}^r(t_i)$  upon which it plans its trajectory to improve WET efficiency.

#### A. Underwater Wireless Energy Transfer

Underwater WET using magnetic induction has been widely implemented and tested [2], [3]. The US Department of Energy has identified underwater WET as an important technology for marine energy [3]. In existing underwater WET, AUVs have to return to a charging station or to be docked on top of a mobile charger; the relative positions of the mobile charger and AUVs are static. This paper considers a radically different scenario where AUVs' are not interrupted by WET, except for sending their location and battery information, and the mobile charger plans its trajectory to optimally charge AUVs.

It was shown in [4], [5] that the effect of orientation loss of magnetic induction can be overcome by using tri-axis coils. Also, the received power can be increased by at most 5 dB by using channel state information [5]. However, it is challenging to perform channel estimation for AUVs due to their mobility. Hence, in this paper we use tri-axis coils without channel state information for WET. The received power can be written as [4], [5]

$$\tilde{P}_{r}(\boldsymbol{x}^{r}, \boldsymbol{x}^{m}) = \frac{\omega^{2} \Lambda_{1}^{2} P_{t}}{32 r_{c}^{2}} \left[ \left( r^{-6} + \frac{|k_{2}|^{4}}{4} r^{-2} \right) e^{-2k_{2}^{im} r} + \left( k_{1}^{2} r^{-4} + r^{-6} \right) e^{-2k_{2}^{im} (d_{r} + d_{m})} \right], \quad (3)$$

where  $\Lambda_1 = \mu_2 \pi a^4 N_c^2$ ,  $\mu_2$  is water permeability, a is coil radius,  $N_c$  is the coil number of turns,  $r_c$  is the coil resistance,  $d_r$  is the depth of the AUV,  $d_m$  is the depth of the mobile charger,  $P_t$  is the transmission power,  $r = ||\mathbf{x}^r - \mathbf{x}^m||_2$ ,  $k_1$  is the propagation constant of the air, and  $k_2^{im} = \Im(k_2)$  which is the imaginary part of the propagation constant of water  $k_2$ . Note that, the above equation assumes the mobile charger and the AUV are separated by a certain distance to avoid collisions and the strong coupling effect is neglected. However, when their distance is small, this model can give infinitely large received power due to its singularity. To reduce it effect, we employ a modified received power which is  $P_r(\mathbf{x}^r, \mathbf{x}^m) =$  $0.5\tilde{P}_r(\mathbf{x}^r, \mathbf{x}^m)P_t/[0.5P_t + \tilde{P}_r(\mathbf{x}^r, \mathbf{x}^m)]$ . In this way, when the distance is small, the efficiency cannot be higher than 50% which is considered as the highest efficiency.

# B. Single AUV

First, we study the WET for a single AUV to understand the fundamental challenges. The mobile charger maintains an estimator of the AUV's location by using the Kalman Filter [6]. Once the mobile charger receives  $y^r(t_i)$ , it uses Kalman Filter to estimate current location  $\hat{x}^r(t_i)$  and predict  $\hat{x}^r(t_{i+1})$ . To plan the mobile charger's trajectory, we divide the time slot between  $t_i$  and  $t_{i+1}$  into  $n_t$  subslots with  $t_i^j = j(t_{i+1} - t_i)/n_t$ , for  $j = 0, 1, \dots, n_t - 1$ , and consider its velocity in each time slot as a constant. This approach has been widely adopted in terrestrial robot trajectory planning [7]. As a result, the predicted AUV's velocity is  $\hat{v}^t(t_i)$  and location is  $\hat{x}^r(t_i) + t_i^j \hat{v}^t(t_i)$ , for  $j = 1, \dots, n_t$ .

 $t_i^j \hat{\boldsymbol{v}}^t(t_i)$ , for  $j = 1, \cdots, n_t$ . Note that  $\frac{d\tilde{P}_r(\boldsymbol{x}^r, \boldsymbol{x}^m)}{dr} < 0$  which indicates that the received power is monotonically decrease as the distance increases and the distance should be as small as possible to increase the received power. Assume the minimum required distance between the mobile charger and an AUV is  $r_{min}$  to avoid collisions. Then, the optimal trajectory for the mobile charger is to keep  $r_{min}$  away from the AUV and there are infinite solutions. To simply the motion of the mobile charger, we consider it follows the AUV with the trajectory  $\hat{\boldsymbol{x}}^r(t_i) + t_j^j \hat{\boldsymbol{v}}^t(t_i) - r_{min} \hat{\boldsymbol{v}}^t(t_i) / \| \hat{\boldsymbol{v}}^t(t_i) \|_2$ , for  $j = 1, \cdots, n_t$ .

# C. Multiple AUVs

Next, we consider a swarm of AUVs with locations  $x_1^r(t_i), \dots, x_{n_{rb}}^r(t_i)$ , velocities  $v_1^r(t_i), \dots, v_{n_{rb}}^r(t_i)$ , and battery levels  $b_1(t_i), \dots, b_{n_{rb}}(t_i)$ , where  $n_{rb}$  is the number of AUVs in the swarm. AUVs' energy consumption model is complicated because it consists of various aspects, including locomotion, communication, computation and sensing. Hence, AUVs' energy consumption is hard to predict since it depends on the specific application and underwater dynamics. In this paper, we use a linear model to approximate the energy consumption, i.e.,  $E_c(t_{i+1}, t_i) = h_e(t_{i+1} - t_i)$ , where  $h_e$  is the energy consumption rate and it varies depending on the specific application. Thus, the battery level change of the *l*th AUV during one time slot is

$$b_{l}(t_{i+1}) = b_{l}(t_{i}) + \Delta t \left[ \frac{1}{n_{t}} \sum_{j=1}^{n_{t}} P_{r}^{j}(\boldsymbol{x}_{l}^{r}(t_{i}^{j}), \boldsymbol{x}^{m}(t_{i}^{j})) - h_{e} \right],$$
(4)

where  $P_r^j(\boldsymbol{x}_l^r(t_i^j), \boldsymbol{x}^m(t_i^j))$  is the *l*th AUV received power at time  $t_i^j$ . We implicitly assume that the received power in a subslot is a constant.

To design sustainable underwater cyber-physical systems, the mobile charger seeks to maximize the battery levels of AUVs which is equivalent to maximizing the received power. In this paper, we consider the following two key problems. First, the AUV with the minimum battery level should be charged first to maintain the AUV's normal function. In this case, the mobile charger has to pay attention to a single AUV, which may sacrifice the swarm's overall received power. Second, if the AUVs have similar battery levels, the mobile charger has to consider the overall charging efficiency of the swarm. Thus, there is a tradeoff between the individual's benefit and the swarm's benefit. With this in mind, given each AUV's predicted location  $\boldsymbol{x}_{l}^{r}(t_{i}^{j})$ , we have the following problem:

$$(P1): \max_{\boldsymbol{x}^{m}(t_{i}^{j})} \sum_{j=1}^{n_{t}} \alpha^{j} + \frac{\beta \sum_{l=1}^{n_{rb}} b_{l}(t_{i+1})}{n_{rb} b_{max}}$$
(5a)

s.t. 
$$b_l(t_i^j)/b_{max} \ge \alpha^j, l = 1, \cdots, n_{rb}, j = 1, \cdots, n_t;$$
 (5b)

$$\|\boldsymbol{x}^{m}(t_{i}^{j}) - \boldsymbol{x}_{l}^{r}(t_{i}^{j})\|_{2} \ge r_{min}, l = 1, \cdots, n_{rb}, j = 1, \cdots, n_{t};$$
(5c)

where  $b_{max}$  is the maximum battery level. In (5a),  $\alpha$  is used to improve the minimum battery level to ensure fairness of wireless charging. When the battery level of an AUV is extremely low,  $\alpha$  is a small number which can create a large penalty. The second term in (5a) is used to improve the overall received power of the swarm when AUVs have similar battery levels. Without the second term, the mobile charger may keep moving around to charge the AUV with a minimum battery level which can significantly reduce its efficiency. The battery level can be found by using (4). Equation (5c) is used to ensure that the minimum distance between the mobile charger and an AUV is larger than  $r_{min}$ .

The above problem cannot be solved efficiently since it is nonconvex. Next, we reformulate the problem based on the following two observations. First, WET efficiency decreases fast as the distance increases. Only if the AUV is close to the mobile charger, it can receive significant power. Thus, when the distance between two AUVs is large, the mobile charger may choose to charge one AUV to maximize the overall received power. This will be evaluated in the numerical analysis. Second, the received power is a monotonically decreasing function of the distance between the AUV and the mobile charger. Therefore, to increase the received power, we need to reduce the distance. Based on the above observations, we formulate a new problem which is

$$(P2): \min_{\boldsymbol{x}^m(t_i^j)} \sum_{j=1}^{n_t} \alpha^j$$
(6a)

s.t. 
$$\frac{b_{max}}{b_l(t_i)} \| \boldsymbol{x}^m(t_i^j) - \boldsymbol{x}_l^r(t_i^j) \|_2 \le \alpha^j;$$
(6b)

$$\|\boldsymbol{x}^{m}(t_{i}^{j}) - \boldsymbol{x}_{l}^{r}(t_{i}^{j})\|_{2} \ge r_{min}, \qquad (6c)$$

$$l = 1, 2, \cdots, n_{rb}, j = 1, 2, \cdots, n_{t}.$$

The problem aims to reduce the distance between the mobile charger and AUVs to increase the received power. Also, we consider fairness by using  $\frac{b_{max}}{b_l(t_i)}$ , i.e., if an AUV's battery level is low, the mobile charger moves towards it. Note that, the constraints in (6c) are nonconvex. To relax this constraint by using convex functions, we adopt the approach in [8]. Then, (6c) can be approximated by

$$[\boldsymbol{x}^{m}(t_{i}^{j-1}) - \boldsymbol{x}_{l}^{r}(t_{i}^{j})]^{T}[\boldsymbol{x}^{m}(t_{i}^{j}) - \boldsymbol{x}_{l}^{r}(t_{i}^{j})] \\ \geq r_{min} \|\boldsymbol{x}^{m}(t_{i}^{j-1}) - \boldsymbol{x}_{l}^{r}(t_{i}^{j})\|_{2}.$$
(7)

The approximation is based on the observations that if at time  $t_i$  there are no collisions and the constraint (5c) is satisfied, we can divide the space by using a plane formed by  $\boldsymbol{x}^m(t_i^{j-1})$  and  $\boldsymbol{x}_l^r(t_i^j)$ . The mobile charger is allowed to move in the half space without the AUV at time  $t_{i+1}$ . Now, the problem (P2) has been approximated by using convex functions and it can be solved efficiently to obtain the optimal trajectory of the mobile charger.

TABLE I SIMULATION PARAMETERS.

Symbol	Value	Symbol	Value	Symbol	Value
a	0.1 m	N <sub>c</sub>	10	$r_c$	0.176 Ω
$\Delta t$	1 s	$P_t$	100 W	$h_e$	2.368 J/s
$b_{max}$	34 kJ	$n_t$	5	$r_{min}$	0.5 m
$f_c$	1 MHz	$\boldsymbol{x}^m(t_0)$	[0;-1;-0.75]	Niter	20
$\epsilon_w$	$81\epsilon_1$	$\mu_2$	$\mu_1$	σ	0.05 S/m



Fig. 2. An example of trajectory tracking using the Kalman filter. The observations are used to estimate the real location of an AUV and predict its motion.

Although the mobile charger can navigate to reduce its distance to AUVs to improve WET efficiency, it may fail in a special case where AUVs are far from each other with similar battery levels. By solving P2, the mobile charger will navigate to the center of the swarm, but the distance to AUVs are large. As we have discussed, if the distance is large, the WET efficiency is extremely small. In this case, it is more efficient to follow one AUV instead of trying to charge multiple AUVs simultaneously which is considered by the second term in (5a). To address this issue, the mobile charger evaluates its distances to AUVs at  $t_i$  and obtains the minimum distance  $d_{min}$  and the maximum distance  $d_{max}$ . Next, if all the AUVs have sufficient energy, i.e., the minimum battery level is larger than a threshold, the battery level of the AUV with the minimum distance is scaled by  $r_{min}/d_{min}$ , otherwise, the  $b_l$  of the AUV with the minimum battery level is scaled by  $r_{min}/d_{min}$ . In this way, if  $d_{min}$  is close to  $r_{min}$ , the mobile charger can charge multiple AUVs simultaneously, whereas if  $d_{min}$  is much larger than  $r_{min}$  and some AUVs do not have sufficient energy, the mobile charger will move towards the one with the lowest battery level and stay close to it.

# **III. NUMERICAL ANALYSIS**

In this section, we numerically analyze the wireless charging performance. The simulation parameters are given in Table I. For the AUV motion model, we consider its velocity is a constant  $[1,1,0]^T$ . Thus, A is a diagonal matrix except for the fourth element in the first row and the fifth element in the second row being  $\Delta t$ . Q and R are diagonal matrices with diagonal elements 0.01. An example of the trajectory tracking is shown in Fig. 2. The initial location of the AUV is [0, -0.5, -0.5]. As we can see, the estimated location is more smooth than the observed location and it is close to the real location of the AUV which can be used to plan the mobile charger's trajectory.

Since the mobile charger's trajectory planning is based on the predicted AUV locations,  $\Delta t$  significantly affects the prediction accuracy. If  $\Delta t$  is large, the mobile charger may lose track of the AUV and the WET efficiency becomes extremely low. However, a small  $\Delta t$  incurs large communication overhead. In Fig. 3, we show the tradeoff between communication



Fig. 3. Effect of the location updating interval  $\Delta t$  on the AUV received power.



Fig. 4. Received power tradeoff between two AUVs. The mobile charger is located on the line that connects the two AUVs.

overhead and WET efficiency. We consider a mobile charger to follow an AUV based on the predicted location. When  $\Delta t$  is 1 s, the AUV's received power is high and stable, whereas as it increases, the received power decreases and becomes unstable due to the low accuracy of the location prediction. Since the  $\Delta h$  is 2.368 J/s, the battery level can be increased if the received power is higher than this. We can further improve the WET efficiency by using larger coils and higher transmission power.

When there are multiple AUVs, there is a tradeoff among the charging efficiency for different AUVs. Here, by using two AUVs, we show the effect of this tradeoff. We consider the mobile charger is on the line that connects the two AUVs, and its location can be any point on the line provided that it satisfies (5c). Here, we consider the AUVs to be static. The distance between the two AUVs is 1.5 m and 3 m. As shown in Fig. 4, when they are close, both of them receive reasonable power, whereas when they are far away, if AUV 2 receives a large power, then AUV 1 receives negligible power. Thus, when the distance between AUVs is small, the mobile charger can simultaneously charge multiple AUVs. On the contrary, if the distance is large, it is better for the mobile charger to charge only one AUV and stay close to it to improve the overall WET efficiency.

Next, we consider there are two AUVs in a swarm for better exposition. The two AUVs have the same initial battery level which is  $0.5b_{max}$ . We first generate the trajectory of AUV 1, then we shift the location by  $[0, -y_s, 0]$  to generate the trajectory of AUV 2, where  $y_s = 1.5$  m for the first 10s and  $y_s = 6$  m for the second 10s. As we can see from Fig. 5, the mobile charger is in the middle of the two AUVs when they are close, whereas it moves close to AUV 2 when their distance is large. In the second 10s, it is not efficient for the mobile charger to stay in the middle since this significantly reduces the charging efficiency. The received power during this period is shown in Fig. 6. When AUVs are close, both



Fig. 5. Mobile charger's trajectory for charging two AUVs with the same initial battery level. For the first 10 seconds, the two AUVs are separated by 1.5 m. For the second 10 seconds, the AUVs are separated by 6 m.



Fig. 6. Received power of AUVs associated with the trajectories shown in Fig. 5.

of them receive high power. When they are widely separated, one receives much more power than the other one.

### IV. CONCLUSION

In this paper, we study the problem of underwater mobile wireless charging for swarm robotics. We develop trajectory planning algorithms for charging a single AUV and multiple AUVs in a swarm. We find that the mobile charger needs to stay close to AUVs when the swarm is compact, whereas it has to chase a single AUV when the swarm is scattered. Our solution is based on the direct relationship between the received power and distance. In our future work, we will consider the optimal signal transmissions by using tri-axis coils when AUVs are moving to further improve the wireless energy transfer efficiency and develop efficient solutions to maximize the overall received power of a swarm.

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